Far Field Projection in Tidy3D

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Abstract

Given the computational cost of running a full FDTD simulation, it's often desirable to calculate field data at points outside the simulation domain without expanding the simulation domain to do a full-on solve. Tidy3D's field projection calculates reliable far-field approximations from near-field FDTD data based on fundamental physics principles.

The idea is the following:

1. Record near field data on a projection surface from FDTD
2. Calculate the field data's equivalent current densities on this surface
3. Calculate the far field from these equivalent current densities

1 Projection Surfaces & the Electromagnetism Uniqueness Theorem

Projection surfaces provide an easy way to package field data from complicated systems into data we can easily project. The underlying physics behind this projection is the **Electromagnetism Uniqueness Theorem**, and the resulting **Surface Equivalence Principle**.

Electromagnetism Uniqueness Theorem

Consider a volume V bound by surface S. A solution to Maxwell's Equations, if it exists, is unique if the initial values are specified throughout all of V, and the E or H component tangential to S are specified at all times. In practice, the surfaces we use are Tidy3D Field Projection monitors.

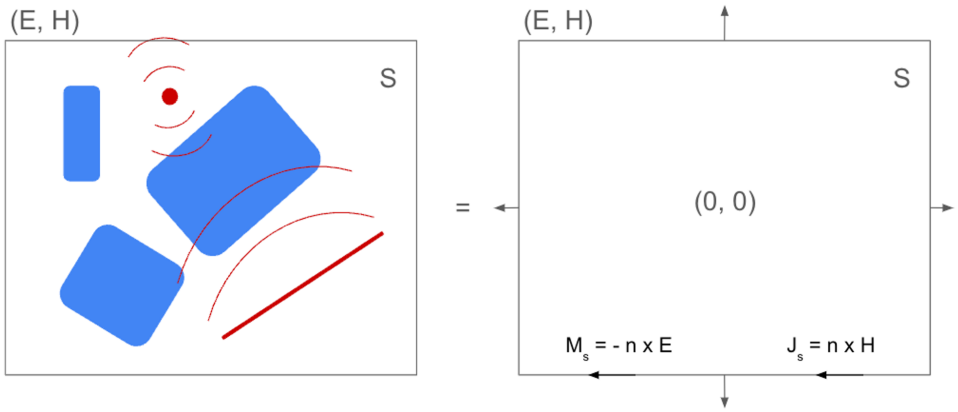
2 Equivalent current densities

Surface Equivalence Principle

Consider a system with arbitrary structures and sources contained in an imaginary volume V with bounding surface S. Label the electric and magnetic fields outside of V by E and H. Due to the Uniqueness theorem, we can define electric and magnetic currents on S such that the electric and magnetic fields inside V are 0, and E and H are the same outside V:

$$J_s = \hat{n} \times H, \; M_s = -\hat{n} \times E$$

where $\hat{n}$ is the unit normal vector to S.



By defining these currents on the surface, we are defining the tangential components of the fields to S at all times, and thus we get the same solutions as the original system. As we will see subsequently, J_s and M_s make it much easier to calculate the far field.

NOTE: in order for these surface currents to be used, either 1) the surface needs to be closed, or 2) the field strength must decay to 0 toward the edges of the surface. When a surface isn't closed, a windowing function may need to be applied to induce such a decay. This can be set in the `window_size` attribute of Tidy3D Field Projection monitors.

3 Calculate Far-Field

To suitably consider fields a large distance away from our surface (and thus original system), we return to Maxwell's Equations. We assume there are no materials with polarization and magnetization, that ϵ and μ are constant in this far region, and we assume the E and H fields have a time dependence of $e^{j\omega t}$. Then, Faraday's and Ampere's Laws become

$$\nabla \times E = -i\omega\mu H = -i\omega\nabla \times A$$
$$\nabla \times H = J + i\omega\epsilon E$$

where A is the magnetic vector potential. Therefore, since $\nabla \times (E + i\omega A) = 0$, E and $i\omega A$ differ by the gradient of some scalar function Φ. Combining this with our new

Ampere's Law, we have

$$\nabla \times H = J + i\omega\epsilon(-i\omega A - \nabla\Phi)$$

Taking the curl of this, and remembering our $\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$ identity, we get

$$\begin{aligned} \nabla(\nabla \cdot A) - \nabla^2 A &= \mu J + \omega^2 \mu\epsilon A - i\omega\mu\epsilon\nabla\Phi \\ \Rightarrow \nabla^2 A + \omega^2\mu\epsilon A &= -\mu J + \nabla(\nabla \cdot A + i\omega\mu\epsilon\Phi) \end{aligned}$$

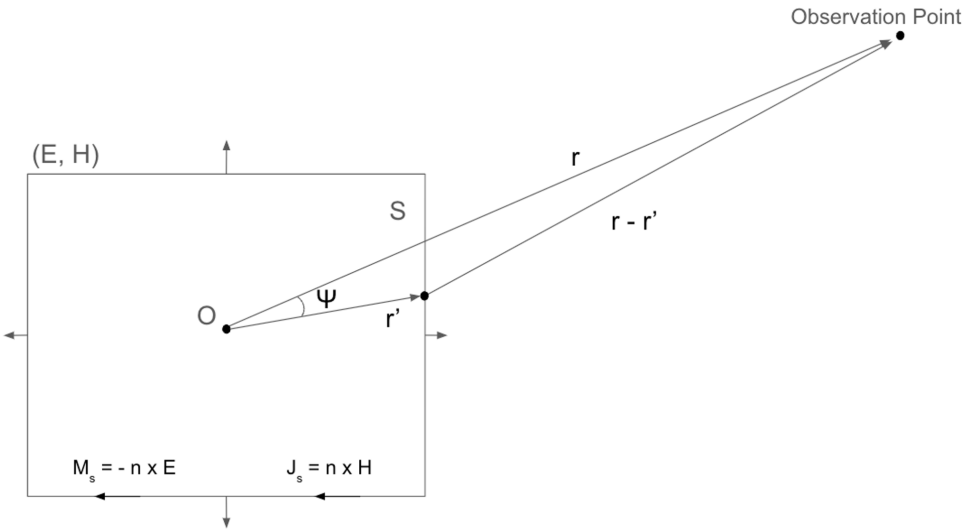
Since we can set the divergence of A to anything we want, we'll set it to $-i\omega\mu\epsilon\Phi$. Following the same procedure for magnetic current M and electric vector potential F results in

$$\begin{aligned} \nabla^2 A + \omega^2\mu\epsilon A &= -\mu J \\ \nabla^2 F + \omega^2\mu\epsilon F &= -\epsilon M \end{aligned}$$

This equation gives us a nice way to calculate the far field from J. When there is an infinitesimal surface current σds at the origin, the above vector potential equation becomes

$$\begin{aligned} \nabla^2 A + k^2 A &= -\mu\sigma ds\delta(r) \\ \Rightarrow A &= \frac{\mu\sigma ds}{4\pi}\frac{e^{-ikr}}{r} \end{aligned}$$

where we simplify by defining $k = \omega\sqrt{\mu\epsilon}$.



Integrating over the entire surface gives our vector potentials at r as

$$A(r) = \frac{\mu e^{-ikr}}{4\pi r} \oint_S J_s e^{ikr' \cos\Psi} ds', \; F(r) = \frac{\epsilon e^{-ikr'}}{4\pi r} \oint_S M_s e^{ikr' \cos\Psi} ds'$$

where r' is a vector from S to r, and Ψ is the angle between r and the point on S. From here, we can calculate our E and H fields via

$$E(r) = -i\omega(A(r) + \frac{1}{k^2}\nabla(\nabla \cdot A(r))) - \frac{1}{\epsilon}\nabla \times F(r)$$
$$H(r) = -i\omega(F(r) + \frac{1}{k^2}\nabla(\nabla \cdot A(r))) + \frac{1}{\mu}\nabla \times A(r)$$

Keep in mind that we've done no approximations up to this point. If our FieldProjection Monitors have `far_field_approx` set to `False`, we simply return these fields. If `True`, we do far-field approximation, neglecting terms that decrease faster than, or on the order of, $\frac{1}{r^2}$. This speeds up our calculations, but, obviously, decreases accuracy.

Field projection data can be returned in polar coordinates: $E_r(r)$, $E_\theta(r)$, $E_\phi(r)$, $H_r(r)$, $H_\theta(r)$, and $H_\phi(r)$.

Tidy3D offers three ways of defining the observation points at which we'll cal-

culate the far field. The observation points can be defined in cartesian, polar, or k-space coordinates, relative to the `custom_origin` attribute. These monitors are the FieldProjectionCartesianMonitor, FieldProjectionAngleMonitor, and the FieldProjectionKSpaceMonitor, respectively.

To see these principles in action, check out our Tidy3D documentation on field projection:

API references

FieldProjectionCartesianMonitor

FieldProjectionAngleMonitor

FieldProjectionKSpaceMonitor

Example notebooks

General Usage

Field projection in cartesian coordinates - Fresnel lens

Field projection in polar coordinates - plasmonic nanoparticle

Field projection in k-space - photonic crystal L3 cavity

Acknowledgements

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References

[1] A. Taflove, S.C. Hagness, M. Piket-May. "Computational electromagnetics: the finite-difference time-domain method." The Electrical Engineering Handbook 3 (629-670), 15

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